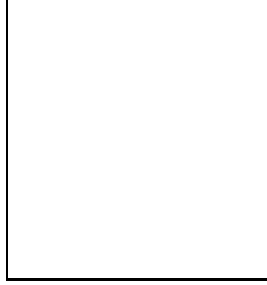


GALACTIC HYDRAULIC DROP?

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Galaxies and galaxy clusters are observed to have a rather non-trivial radial behaviour. The observations show that the radial profiles change from one power-law profile near the centre to another power-law profile in the outer region. We present a simple explanation for this complex behaviour by finding the analytical solutions to the governing hydrodynamic equations. We see that the origin of this complexity is the collisional nature of the baryonic plasma, possibly related to a turbulence-enhanced viscosity.

1 Introduction

Large gaseous baryonic structures such as galaxies and galaxy clusters have been known and observed for many years. A characteristic behaviour is that the radial profiles, e.g. of surface brightness or electron density, often have the complex behaviour that they follow one power-slope, α , in the inner part, and another power-slope, β , in the outer part beyond a characteristic radius, r_0 ,

$$A = \frac{A_0}{\left(\frac{r}{r_0}\right)^\alpha \left(1 + \frac{r}{r_0}\right)^\beta}, \quad (1)$$

and this transition is frequently observed to be rather sharp. This behaviour is often fit by observers by simple phenomenological profiles like eq. (1) which is composed of just such two power-laws. These observations include spiral galaxies (e.g. using WFPC2 data^{1,2}) and clusters of galaxies (e.g. using Chandra data³). However, there is little (if any) theoretical guidance for the use of such profiles. Here we attempt a derivation of this complex behaviour.

Traditionally one would expect that two different power-slopes must be related to different physics, in particular if the transition is sharp. Surprisingly enough, for the radial density profile this does not have to be the case. We will show that the governing equations have exactly two solutions, which imply that the inner and outer density profiles may choose different solutions, and hence quite generally will be different (see ref. [4] for details).



Figure 1: The *circular hydraulic jump*, as observed in any kitchen sink. The water chooses one solution inside the jump and another solution outside the jump. We show that galaxy clusters behave in a similar manner, and that the density profile (or surface brightness) therefore is expected to exhibit a break at some characteristic radius, providing us with a *galactic hydraulic drop*.

In the field of hydrodynamics cases are known where a given set of equations have two solutions, and that Nature chooses to use *both* solutions simultaneously. One well-known example is the *hydraulic jump*, which is a centimetre large ring, observed in any kitchen sink when the water flows out radially after hitting the sink (see Figure 1). The water in the inner few centimetre follows one solution, and outside the jump the water follows another solution^{5,6}. It turns out that in a similar manner the density profile in the inner part of e.g. a relaxed galaxy cluster chooses one solution, whereas the outer part of the same cluster chooses another solution.

2 Solving the Navier-Stokes equations analytically

The behaviour of any collisional gas or fluid is fully determined by the Navier-Stokes (N-S) equations. Baryons often have sufficient collisions to be described by the N-S equations, e.g. in a typical intra-cluster gas the equilibration timescale is about 10^7 years, with mean free path of tens of kpc compared to radii of few Mpc.

In this proceeding we will for simplicity consider a stable spherical cluster of galaxies, where the system has picked out an orientation in space, such that all the gas is moving only in the Θ -direction. Thus we have $v_r = v_\phi = 0$. Here we use notation where r is the radial coordinate, Θ is the angle in the xy -plane, and ϕ is the angle from the z -axis. One must keep in mind that by considering the N-S equations we are taking a fluid approach which implies that we are following a fluid element, and this basically corresponds to averaging over all the particles moving through that fluid element. For the Θ -velocity we consider the simple form

$$v_\Theta = v_\alpha \left(\frac{r}{r_\alpha} \right)^\alpha \sin\phi, \quad (2)$$

where α is the constant to be determined first, v_α and r_α are unknown constants, with the physical interpretation that r_α is a characteristic transition radius, and v_α is the velocity of the fluid element at that radius.

The first N-S equation. The first N-S equation becomes very simple with the assumed form of the velocities

$$0 = \nu \left[\nabla^2 v_\Theta - \frac{v_\Theta}{r^2 \sin^2 \phi} \right], \quad (3)$$

where ∇^2 is the scalar Laplacian, and ν is the kinematic viscosity. For now all that matters is the existence of a non-zero viscosity, so the absolute magnitude (and even radial dependence) is not important for the results. Certainly baryons have non-zero viscosity, however the viscosity could also arise from turbulence, in which case it could be very large⁸, $\nu_{\text{turb}} \sim l\Delta v$, where the dimension l is the size of the turbulent eddies, and Δv is the velocity dispersion. Numerical simulations⁷ have shown that such turbulence indeed exists in galaxy clusters with $\Delta v \sim 300 - 600$ km/sec, and $l \sim 100 - 500$ kpc, leading to a very large (turbulence enhanced) viscosity.

When we use the form for v_Θ in eq. (2), then eq. (3) has exactly 2 solutions

$$\alpha = 1 \quad \text{or} \quad -2. \quad (4)$$

Thus, looking at eq. (2), it is clear that the solution with $\alpha = -2$ is divergent for $r \rightarrow 0$, and we will therefore refer to this solution as the '*outer solution*', and similarly, the solution with $\alpha = +1$ is inconsistent for large radii, and we will refer to this as the '*inner solution*'. Hence the general flow pattern changes from the inner to the outer region, and one may therefore expect to find different density profiles in the central and outer regions.

The second N-S equation. We can now use the next N-S equation to extract the asymptotic radial density profiles. Also this equation is very simple

$$-\frac{v_\Theta^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{M(r)G}{r^2}, \quad (5)$$

where ρ is the radially dependent density, P is the pressure, G is the gravitational constant, and $M(r)$ is the mass within the radius r . We assume that the pressure and density are related through $P = P_\alpha (\rho/\rho_\alpha)^\gamma$, where P_α and ρ_α are the unknown pressure and density at r_α . We take a monatomic gas with $\gamma = 5/3$. Let us consider densities of the form

$$\rho(r) = \rho_\alpha \left(\frac{r}{r_\alpha} \right)^\beta, \quad (6)$$

such that the parameter β determines the density profile. It is worth emphasizing that it is exactly this β which we are trying to find.

The last (gravitational) term including $M(r)$ depends on the given system we are considering. If the mass is dominated by a point gravitational source (e.g. a central black hole (BH)), then it goes like $M(r)G/r^2 \sim r^{-2}$. If the mass is dominated by the matter density, then it goes like $M(r) \sim \int \rho(r)dV$, with dV the volume element. For spherical solutions this gravitational term thus goes like $r^{\beta+1}$ with β from eq. (6).

It is now straight forward to solve eq. (5) in the inner and outer regions. The $\alpha = 1$ is the inner solution, for which we find $\beta = -6$ when the baryons dominate the mass. With BH dominance one has $\beta = -3/2$, and if dominated by another spherical distribution (which probably should arise from dark matter (DM)) with profile $\tilde{\beta}_s$, then we find $\beta = 3/2(\tilde{\beta}_s + 2)$. For a different polytropic index, $1 < \gamma < 5/3$, the coefficient changes from $-3/2$ to $(\gamma - 1)^{-1}$. Thus, if the DM has a slope of $\tilde{\beta}_s = -2$ (as expected from adiabatic contraction), then the baryons develop a core, $\beta = 0$.

In the outer region ($\alpha = -2$) we find $\beta = -6$. If a BH dominates then $\beta = -3/2$, and again if a another spherical distribution (DM) dominates then $3/2(\tilde{\beta}_s + 2)$. E.g. a DM slope of -3 leads to $\beta = -3/2$. One should, however, keep in mind that in the outer region there are possibly not sufficient collisions to assure the validity of the N-S equations, so those simple solutions should not be trusted too much.

3 How to test these findings

Our main finding is that a transition from the inner to the outer region generally exists. This is observationally well established^{1,2,3}. We have thereby provided theoretical support for the use

of phenomenological profiles like eq. (1). One can now take a further step and test the actual numbers we find in our simplified treatment.

The findings for BH domination always give $\beta = -3/2$, which is just what numerical simulations find¹¹. The actual density profiles can be observed in different ways. X-ray observations of the luminosity in various bands give the electron density of the plasma as a function of radius. E.g. for the relaxed cluster A2029 the outer baryonic profile of $\beta = -1.62$ was found³. In the future the Sunyaev-Zeldovich effect will directly provide a measure of the radial electron density^{12,13}, which will probe large cluster radii since the SZ effect is proportional to n_e whereas X-ray observations are proportional to n_e^2 . Surface brightness from radio observations of *HI* and molecular gas can in principle determine the baryon profile¹⁴. For details and discussion see ref. [4]. We are looking forward to doing a more detailed analysis.

4 Conclusions

We have presented an explanation for the origin of the complex radial structure of galaxy clusters. Specifically, we have shown that the density profiles generally are expected to make a transition from one power-slope in the inner to another power-slope in the outer region

$$\rho_{\text{gas}}(r) = \frac{\rho(0)}{r^{\beta_1}(1+r)^{\beta_2}}. \quad (7)$$

The physical origin of this complexity is the collisional nature of the baryonic plasma, and we speculate that it may be related to a turbulence-enhanced viscosity.

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